# Adjoint Monte Carlo Method for the Multiplicity Moment Distributions of Subcritical Multiplying Systems

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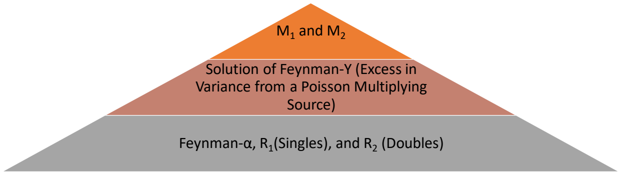
## Introduction:

- **Correlated neutron counting is a popular family of techniques that aims to characterize fissile material systems.**
- **We want:**




- **The two biggest challenges faced when employing these methods are the low-fidelity of the solution to spectral and geometric effects as well as well as the long-runtimes when processing list-mode data in MCNP.**

Experiment using the BeRP (beryllium reflected Pu) ball reflected by polyethylene hemispherical shells. These measurements can be used for algorithm and code validation and can be used to validate the adjoint MC solution of Feynman-Y. [1]

## Method:

The method that is being employed is based on work from Gabrieli [2] in their extraction of statistical moments using an adjoint formulation of the stochastic neutron transport equation developed by Munoz-Cobo:

**Adjoint Formulation of Moment Equations**

- $\left(-\frac{1}{v}\frac{\partial}{\partial t} - L^{\dagger}\right) M_{1,T}(\vartheta,t) = \Theta(t)\Theta(T-t)\Sigma_d$
- $\left(-\frac{1}{v}\frac{\partial}{\partial t} - L^{\dagger}\right) M_{2,T}(\vartheta,t) = \overline{\nu(\nu-1)}\Sigma_f(\vartheta)\left(\int \chi_f M_{1,T}\, d\Omega' dE'\right)^2$

**Forward Monte Carlo Approach to Adjoint**

- $\Psi^{\dagger}(\vartheta_n,t_n) = \frac{1}{w_n}\iint_0^T G(\vartheta_n,t_n \to \vartheta,t)\, Q^{\dagger}(\vartheta)\, d\vartheta dt$

**Computing the Feynman-Y**

- $Y(T) = \frac{\iint_{-\infty}^{T} Q(\vartheta) M_{2,T}(\vartheta,t)\, d\vartheta dt}{\iint_{-\infty}^{T} Q(\vartheta) M_{1,T}(\vartheta,t)\, d\vartheta d}$

Source Adjuncton → Initiate Collision Loop → Apply Variance Reduction (Russian Roulette, Implicit Capture) → Register Path Lenth Estimate Score Based on Adjoint Source → Store Historical Data → End History Loop

End History, $\Psi(E_j,t_j) = S(E_j,t_j) / N$

End History, $\Psi(E_i,t_i) = S(E_i,t_i) / N$

End History, $\Psi(E_k,t_k) = S(E_k,t_k) / N$

Intermediate History 1, $\Psi(E_i,t_i) = (S(E_i,t_i) + S(E_j,t_j) + S(E_i,t_i))/ N$

Intermediate History 2, $\Psi(E_m,t_m) = (S(E_m,t_m) + S(E_k,t_k))/ N$

Source Adjuncton, $\Psi(E_0,t_0) = (S(E_0,t_0) + S(E_i,t_i) + S(E_m,t_m))/ N$

## Preliminary Results & Moving Forward:

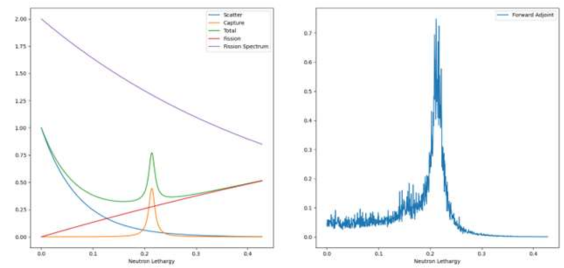
Currently, the MC code can compute the adjoint flux for the 1st and 2nd order moments of multiplicity. These moments are then convolved with the forward neutron source to obtain the Feynman-Y solution. The long-time limit of the Y solution was then used to compute α, the prompt neutron decay constant, which is then used to compute $R_1$ and $R_2$, the singles and the doubles count rates.

Issues:

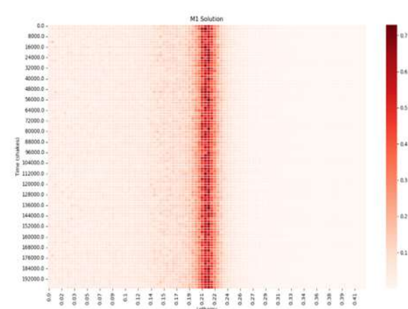
- Order of magnitude between the adjoint and analog solutions of Feynman-Y are different by 7 orders of magnitude.
- The second moment adjoint source isn't telling the full story and needs to consider terms that deal with the detector response itself.

Future Work:

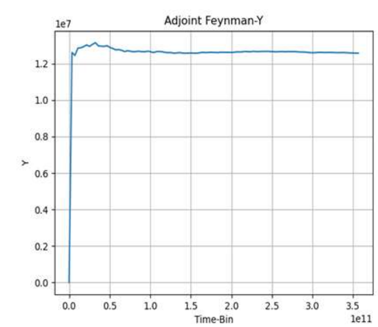
- Compare the adjoint solutions of the moment distributions with the distribution computed analytically with the same system parameters to verify the validity of the method at the proof-of-principle stage.
- Adjust the second adjoint source to consider the detector response.
- Implement the adjoint solver with MCNP to validate the method with experimental data (see BeRP ball).




Left: Plot of the various fictitious continuous cross-sections used to develop toy model utilizing a forward Monte Carlo simulation. Right: Monte Carlo solution obtained from a forward Monte Carlo simulation where the adjoint source is the capture cross section (response function). Forward neutrons around ~0.2 lethargy range contribute most to the response of interest.




Plot representing the adjoint flux solution w.r.t. time in unit shakes and lethargy.




Adjoint solution to the Feynman-Y equation over varying gate-width time bins. This relies on the first and second adjoint statistical moments of multiplicity. Variations at earlier times can be attributed to problem specific parameters such unrealistic cross sections or a second adjoint moment that does have not a fully defined source (in progress).

**References:**

[1] Hutchinson, J., Bounds, J., Cutler, T., Dinwiddie, D., Goda, J., Grove, T., … Walker, J. (2021). A New Era of Nuclear Criticality Experiments: The First 10 Years of Radiation Test Object Operations at NCERC. *Nuclear Science and Engineering*, *195*(sup1), S80–S98. https://doi.org/10.1080/00295639.2021.1918938

[2] G. Gabrieli, I. Neder, U. Steinitz, Full calculation of the long-time-interval limit of noise in reactors, Annals of Nuclear Energy, Volume 191, 2023, 109944, ISSN 0306-4549, https://doi.org/10.1016/j.anucene.2023.109944.

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